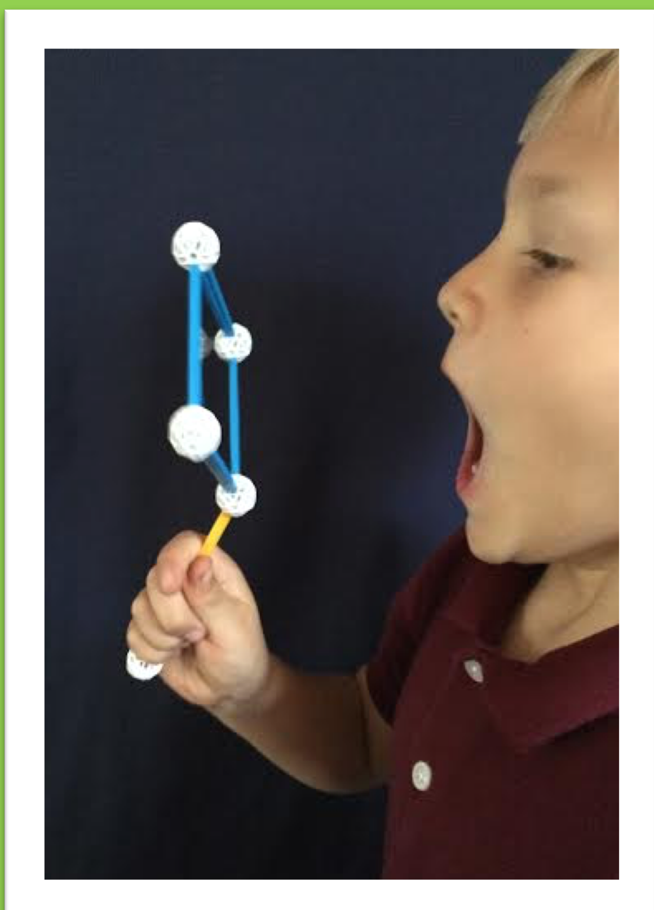
PATTERN SLEUTHS: EXPLORING THE GEOMETRY OF SOAP FILMS & BUBBLES



Grades 6-8

Zometool Inc.
1040 Boston Ave
Longmont CO 80501
United States
Toll Free: 1-855-ZOMETOOL
info@zometool.com

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Pattern Sleuths: Exploring Geometry of Soap Films & Bubbles

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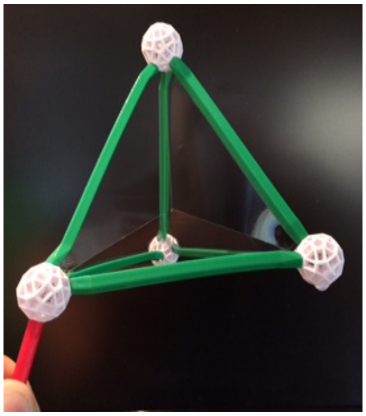
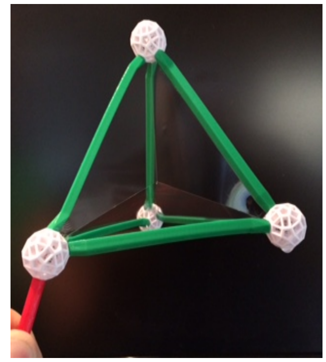


NOTE: *We hope that you enjoy using these materials to supplement your existing units and lessons. Please feel free to duplicate/modify the Student Investigation Guides for use with your students. The remainder of the module materials may not be duplicated or used in professional development without permission. Thank you.*

Exploring the Geometry of Soap Films & Bubbles: Overview



Overview: *Soap bubbles have long captured the curiosity of children and adults alike! But, did you know that these nanostructured toys reflect some of nature's great optimization principles? In this investigation, students will hone their skills as pattern-seekers in exploring the geometrical properties of both bounded and non-bounded soap films and bubbles. They will have the opportunity to create beautiful minimal surface structures, to think like a famous physicist, and to make connections between the patterns of soap bubbles and the patterns of myriad structures in nature.*

Suggested Grade Level: 5th-7th Grade

<table>
<tr><td rowspan="2">Learning Goals</td><td colspan="2">Essential Questions:

What is a pattern? How can patterns help us "see" and understand the structures around us?
How do soap films interact in bounded shapes?
How does nature choose its shapes?</td></tr>
<tr><td>Content Knowledge/Ideas:
Identification of 2D and 3D shapes; Polyhedra characteristics (vertex, edge, face); Surface area
Soap films and bubbles will change shape until reaching a local minimal energy, which is proportional to its minimal, or smallest surface area.
Geometric properties of free-floating bubbles in stable equilibrium: the sphere.
Patterns of line and angle relationships in soap films constrained by a closed boundary (Zometool polyhedron frames): Plateau's Problem</td><td>Focal Skills/Practices:
Students will learn to seek, identify, and represent patterns to better understand and make predictions about the structures around us.</td></tr>
<tr><td colspan="3">Common Core Mathematics Standards: 5.MD.C.3, 6.G.A.4, 7.G.B.6, MP7</td></tr>
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TEACHER CONTENT BRIEF

BASIC ANATOMY OF A ZOMETOOL SOAP BUBBLE: Surface tension broadly refers to the cohesive forces between liquid molecules that keep surface area to a minimum. The surface tension of water, alone, is too strong to make good bubbles. Adding soap reduces surface tension of water, making it more "stretchy." The cohesion between the water molecules in a soap film cause it to constrict into smallest surface area possible (and thus, the lowest energy of the system).

Cross-Field Connections:

While this module focuses on exploring the geometry of soap films and bubbles, these sudsy structures can also be used to explore:

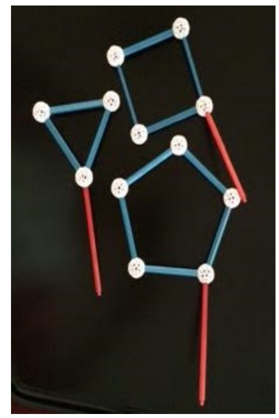
SCIENCE: Molecular interactions and surface tension, Light waves, Air pressure

ENGINEERING: Minimal surface structures and design

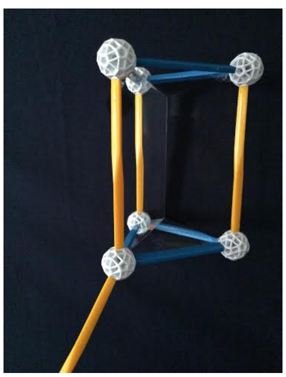
In this module, students will look for patterns in the relationship between differing Zometool polygon "wands" and the resulting bubble shape. Through this investigation, they will learn about one of nature's great economical shapes: the sphere.

WHY ARE FREE-FLOATING SOAP BUBBLES SPHERICAL?

If you have ever tried to blow a bubble from a wand that is not circular, you will find that the resulting free-floating bubbles in stable equilibrium are spherical. When encapsulating a given volume (such as air) under a slightly elevated pressure, the shape that uses the least amount of soap film material – and thus, the minimal surface area and lowest energy of the system, is the sphere.



SOAP FILM PATTERNS OF 3D SHAPES: MINIMAL SURFACES

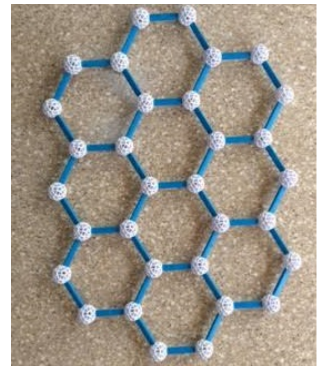


Constrained by a closed boundary (e.g., a Zometool polyhedron model), a soap film stretches to cover the smallest possible surface area, or the minimum surface. That is, soap films look for shapes of minimal surface area and equivalently, minimal energy. In this module, soap films reveal the most efficient way to link the balls and struts within Zometool polyhedra models.

In the 19th century, Belgian physicist Joseph Plateau noticed that a few simple patterns describe the geometry of how soap film surfaces, constrained by a given boundary, meet or interact: 1) three surfaces of a soap film meet at 120 degree angles along an edge, or line and 2) four lines, formed by the intersection of the three surfaces, meet at a vertex; the angle between the adjacent lines is approximately 109.5 degrees, or the tetrahedral angle.

In this module, students will use their knowledge of lines and angles to look for patterns in soap film surfaces bounded by Zometool 3D models. Through this investigation, students think a little like Plateau, arriving at some of his famous principles.

NATURE'S PATTERNS:
From a two-dimensional perspective, bubble structures and honeycomb structures share a common geometric pattern in nature: a network of 120 degree angles that create a hexagonal arrangement with shared walls and no gaps. For a given area, both bubbles and honeycombs minimize their total perimeter (and thus, use the least amount of material). The hexagon is an efficient and economical packing shape. Look for this pattern in the scutes of a turtle carapace, in the facets of a fly's eye, or in mud cracks!

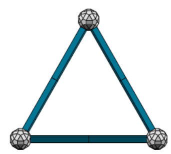


Teacher Toolkit: Zometool Model Examples & Preparation Notes

<u>Zometool Key</u>:

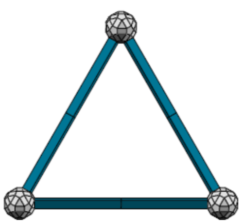
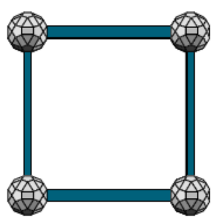
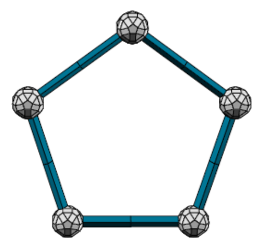
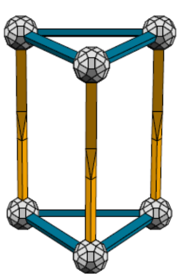
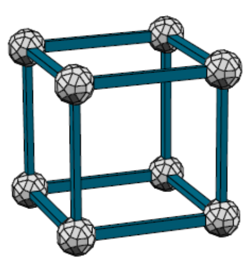
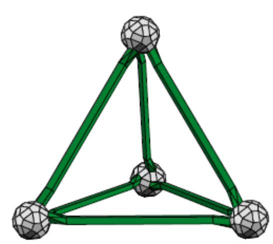
Strut Length:
S=Short
M=Medium
L=Long

Strut Color:
B=Blue Strut
R=Red Strut
Y=Yellow Strut
G=Green Strut



Quantity-(Strut Length)(Strut Color)

3-SB = THREE SHORT BLUE STRUTS needed to make this triangle

Polygon Frames or "Wands"		
3- MB 	4-MB 	5-MB 
Polyhedron Frames		
6-MB & 3-LY 	12-MB 	6-MG 

TEACHER PREPARATION:

SOAP SOLUTION RECIPE: Fill a deep container with warm water with ~2 gallons or 7.5L of water. Add ~2 cups or 500mL of liquid dishwashing detergent (antibacterial products are not recommended) and add one tablespoon of glycerin (available at most drugstores). Mix the solution gently, keeping the surface free of foam. *We suggest setting up 3-4 containers of soap solution. If inside, be sure that there is a drop cloth underneath and surrounding the containers.*

MODULE MATERIALS: *See individual module sections for a breakdown of materials.

• Plastic Circular Bubble Wands (class set) • Team Dry-Erase Boards or Paper/Tape • Large Team Sticky Notes or Paper/Tape • Soap Solution & Containers • Shallow Trays (one per team) • Straws (one per student) • Protractors • Graph Paper (one piece per student) • Copies of Resources A-E	Needed Zometool team kits and models are listed at the beginning of each module section. **TIP: When assembling Zometool team kits for building, be sure to connect each strut to a white node. This will facilitate building.*

Introductory Reason & Engage Prompts

ESSENTIAL QUESTION: W*hat is a pattern? What patterns exist in the structures around us?*

TEACHER PREPARATION:

Suggested Instructional Arrangement: Individual & Teams of 2-3	**Materials:** Student Investigation Guide: Resource A

PART I: Hand-out Resource A to students.

1. Begin with a web brainstorm focused on the broad concept of patterns (see Resource A). Drawing on learners' prior ideas and everyday experiences, challenge students to think about some characteristics and non-characteristics of patterns in our world. Allow students to brainstorm individually and then in pairs or small teams. Monitor student talk and listen for a range of examples and student ideas to highlight.

   Teacher Note: If any students are struggling to get started, have them look at the honeycomb images on the second page of Resource A. *What do you notice?*

2. CLASS DISCUSSION: *What is a pattern*? *What is not a pattern?* Elicit and help student synthesize multiple ideas that provide insight into the concept of a pattern (*potential words might include "repeating, reoccurring, predictable" trait*). Develop a working definition of "patterns" generated by

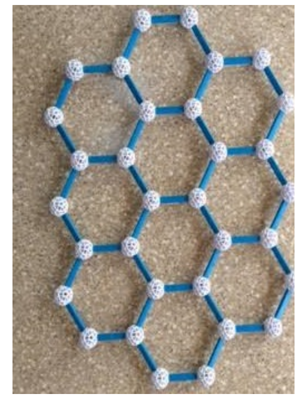
student ideas and reasoning. Prompt students to share their own examples and non-examples, explaining their reasoning.

PART II: Continuation of Resource A

3. Prompt students to look at the visual of a honeycomb and a Zometool model of a honeycomb (you can also make a Zometool honeycomb model using the blue struts for students to view!). Either read the "Did you Know" section to students or have students read this aloud.

DID YOU KNOW? The walls of a honeycomb cell are made of beeswax. Making wax is very expensive process for the bees and requires a tremendous amount energy. Thus, bees are careful to not waste this resource when creating cells that will store honey, nectar, and pollen.

4. PAIRS & CLASS DISCUSSION: Challenge students to:
   - describe the patterns in beehive honeycomb structures (*repeating hexagons without gaps, or a tiling/tessellation*), and
   - think about the reason(s) why beehive honeycombs are designed this way. List all student ideas/reasons in a central location as hypotheses. Encourage students to add or modify these visible class ideas as they work through the next investigations.



5. Point out to students that soap bubble structures and honeycomb structures have some important similarities. In the next investigation, students will continue to hone their skills at pattern-seekers in exploring the following questions: *What patterns can we find in soap structures? What do these patterns tell us about these sudsy structures?*

Investigation I: The Patterns of Soap Bubbles

ESSENTIAL QUESTION: *What patterns can we find in soap bubble structures?*

TEACHER PREPARATION:

Suggested Instructional Arrangements:	**Materials:**
Individual & Teams of 2-3	Plastic Circular Bubble Wands Soap Solution Large Sticky Notes, or Paper and Tape (one per team)

1. Encourage students to close their eyes and imagine a room full of soap bubbles. *What can they recall about the shape, color, feel, etc. of these bubbles?*

2. In teams of 2-3, prompt students to blow bubbles for 1-2 minutes with the circular plastic bubble wands, using an "eagle eye" to make observations of the soap film before and after blowing the bubble.

3. TEAM THINK: On a team sticky note, encourage students to discuss, summarize and write/draw at least 3-4 patterns they found in the soap structures.

4. TEAM SHARE: Have each group share their observations/patterns, posting their sticky notes on a class think board (see below for an example). As each group shares, encourage students to find and talk about both the similarities and differences in whole class observations. *Why do you think this happened? How did you determine this was a pattern? What was your data? I've noticed that many of you have observed that bubbles are round or spherical - is this always the case? How could we find out?* Record potential questions for inquiry that emerge from these conversations. Emphasize the importance of asking questions to dig deeper into our observations!

*Teacher Note: A common idea for "why" bubbles are spherical stems intuitively from the shape of the wand. This is a good hypothesis to highlight – and one that we will explore in this investigation.

Example of a Think Board

THE PATTERNS OF SOAP FILMS & BUBBLES	WHAT WOULD WE LIKE TO FIND OUT?
Bubbles are spherical in shape. Bubbles combine or "stick together" when they meet.	Are free floating bubbles always spherical? Why or why not? What happens when two or more bubbles combine (thought - look at lines and angles)? Why?

*Teacher Note: Throughout the investigations, the think board functions as an excellent place for students to post and share on-going observations, thoughts, and questions.

5. The Anatomy of a Soap Bubble: Using a short video clip, pictures, or even an activity, provide a brief overview of the basic composition of a soap bubble. This will help students make sense of the "why" behind many of the geometrical patterns student will find in upcoming activities.

> *Surface tension broadly refers to the cohesive forces between liquid molecules that keep surface area to a minimum. The surface tension of water, alone, is too strong to make good bubbles. Adding soap reduces surface tension of water, making it more "stretchy." The cohesion between the water molecules in a soap film cause it to constrict into smallest surface area possible (and thus, the lowest energy of the system).*

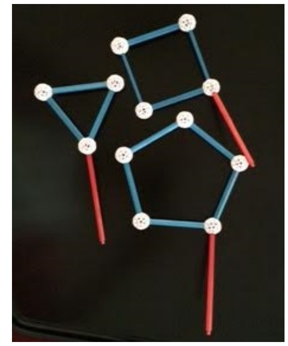
*Teacher note: In the next investigation students will have a chance to unravel some of the most common questions related to the geometry of soap films and bubbles. However, student-derived questions not explored in these lessons can be used in enrichment stations within your classroom, in take-home enrichment activities that extend these ideas, or in later lessons/units.

Investigation II: Shaping Up, Nature's Economical Shape

ESSENTIAL QUESTION: *How does nature choose its shapes?*

OVERVIEW: In this investigation, students will further hone their skills as pattern-seekers by exploring the shape of soap bubbles produced by Zometool polygon wands. *Are free floating bubbles always* *spherical? Is it possible to produce a free floating cube-shaped bubble?* We started with circular bubble wands, but let's see if we can find any patterns between the shape of a polygon frame and the shape of a bubble.

TEACHER PREPARATION:

Suggested Instructional Arrangement: Teams of 2-3	**Suggested Zometool Kit Per Team:** MB: 12 – Each team will make one regular triangle, one square, and one regular pentagon as "polygon wands." MR: 3 – Wand handles *TEACHER NOTE: These can also be prebuilt for students. **Other Materials:** Student Investigation Guide: Resource B Team Dry-Erase Board (or a Large Sticky Note/Piece of Paper) Soap Solution and Containers	

KEY TERM FOR REVIEW: *Surface Area*

1. Distribute the Student Investigation Guide (Resource B). Begin with a Predict-Observe-Explain sequence.

   Predict: Prompt students to build each "polygon wand" (or examine prebuilt models), and draw what they predict will be the resulting shape of the bubble. Pictures of a Zometool regular triangle, square, and pentagon are provided to help facilitate building.

   Observe: In teams of 2-3, have students gently blow bubbles with the three polygon wands at their workspace and make observations. Prompt teams to think about, discuss, and depict *what* happened and *why* it happened on their team dry erase boards. Walk around, observe, and listen. Check in randomly with team members and ask them to summarize the team's thinking and ideas so far. Ask questions that extend thinking or reasoning: *What patterns did you find? What is your evidence for this pattern? Can you draw a picture of what you are thinking? Why do you think that all free-floating bubbles are spherical?*

   Explain: CLASS DISCUSSION - *Is there a relationship between the shape of the polygon frame and the resulting bubble (no, despite the wand shape, all free-floating bubbles are spherical)? How does nature choose its shapes?* Have teams share some of their ideas and reasoning with the class. If needed, scaffold students toward understanding that free-floating bubble pulls itself into a shape that uses the "least amount of material", or the minimum surface area to encapsulate a given volume of air. This shape is a sphere. The sphere is one of nature's great economical or space and energy conserving shapes.

2. Investigation II: Examples of Informal Checks for Understanding

- CLASS CHECK-IN: Return to the Think Board started in Investigation I. With sticky notes, prompt teams to add 3-4 pieces of new evidence, ideas, and/or questions. *What have we learned so far? What new questions have emerged?*

- INDIVIDUAL EXPLANATION (Resource B): Prompt students to develop an explanation *what* happened (i.e., what patterns did they find) and *why* it happened in their own words. Encourage them to support and communicate their reasoning with evidence and a strong focus not only on *what* happened, but *why*. *This is a great place to offer students feedback and a chance to refine the development and communication of their ideas.

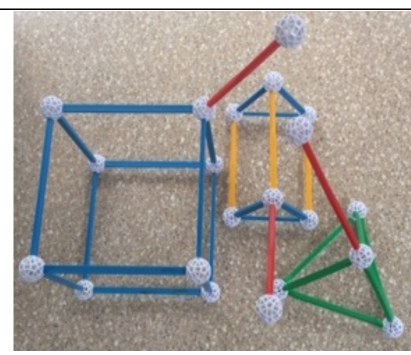
*Teacher Note: Included in Resource B is an "Explanation Checker." However, to be sure that all students extend their understanding and reasoning, challenge learners to extend their a) content ideas and/or b) development and communication of ideas. Examples of feedback questions:
- Can you provide a counter-example here to clarify your ideas?
- Can you represent this idea in more than one way? How could this be communicated more clearly?
- Can you develop an explanation of "why" from a macroscopic and microscopic point of view (i.e., What do you think is happening at a molecular level?).

Investigation III: Exploring Soap Film Patterns in 3D Shapes

ESSENTIAL QUESTION: *What soap film patterns can we find in 3D shapes? How do soap films interact?*

OVERVIEW: In Investigation II, we found that *free-floating bubbles* in stable equilibrium are spheres. This was a non-bounded study of bubbles. In this investigation, however, we will explore soap film patterns formed in shapes constrained by a closed boundary (e.g., Zometool polyhedra models). This is a key distinction to make.

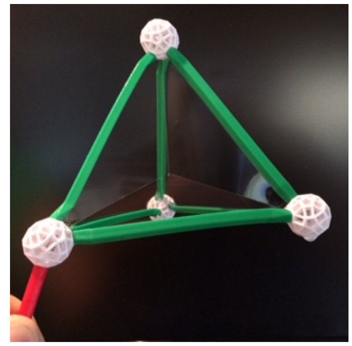
TEACHER PREPARATION:

Suggested Instructional Arrangement: Teams of 2-4	**Prebuilt Models for Teams:** Triangular Prism Cube Tetrahedron *A long red strut can be used as a "wand" **Suggested Zometool Kit** 6-MY **Other Materials:** Student Investigation Guides: Resource C and D Soap Solution and Containers Protractors	

KEY TERMS FOR REVIEW: *Vertex, Edge, Face, Surface Area of a Polyhedron*

1. Distribute Resource C: Exploring Soap Film Patterns in 3D Shapes, and refer students to PART A. Begin with a Predict-Observe-Explain sequence.

PREDICT: Have students individually develop a prediction and explanation of what they think will happen when you dip a tetrahedron into soap solution. *Where will the soap film form?* Pair-share and elicit student thinking. *Who agrees? Disagrees? Why?* Let's see what happens!



OBSERVE: Have teams dip a Zometool tetrahedron completely in soap solution and slowly pull it out. Students should make, draw, and list as many observations as they can. *Where does the soap film form? What shapes are the film faces?* Walk around and listen to conversations. Encourage students to look for patterns in (a) how many soap film faces come together at an edge *(three)*, and the angle measurement here *(120 degree angles)*; and (b) how many lines come together at a vertex *(four).*

As well, students are prompted to build a model of a soap film junction using one white node as a "soap film vertex" and the yellow struts as the lines that come into this vertex. Once a model is built, students can measure the adjacent angles to arrive at the following finding: at a soap film vertex where four lines meet, the angle between the adjacent lines is approximately 109.5 degrees (*also known as the tetrahedral angle*). This model will be helpful for students in Part B.

When ready, teams should move to developing some hypotheses for *why* this happened in the "Explanation" section.

EXPLAIN: At a point that makes sense, bring the class back together to discuss what happened. Have teams share their observation data and key findings. Example of probing questions: *How are these results the same or different from what you predicted? Can you draw a picture of what the soap films look like and how they interact? How did you figure out the angle measurements? What was surprising to you?*

Key Observations to Highlight:

- Soap films did not "cover" the faces of the polyhedron. The soap films pull themselves into the smallest surface area, meeting at the center of the frame. In the tetrahedron, six small soap film faces is less surface area than the four large triangles on the outside. As well, the soap films reveal the most efficient way to link the balls and struts within the tetrahedron.
- Patterns:

How many film faces come together to make an edge? *Three film surfaces come together*	How many lines come together at a vertex? *Four lines*
What is the measure of the angles where you see the films join? *120 degree angles*	What is the approximate measure of the angles you see where the edges come together at the vertex? *Approximately 109 degrees; the tetrahedral angle*

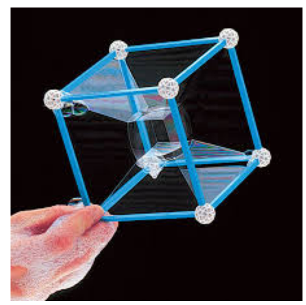
Elicit student ideas for why this may have happened! Let's keep exploring.

2. PART B: Introduce students to Joseph Plateau (1801-1883), a famous physicist. It turns out that Plateau started to see some interesting patterns in how soap film surfaces join and interact in bounded shapes. Point out to students that like Plateau, they need look for and identify patterns in two more polyhedrons: a triangular prims and a cube. *Are you seeing any patterns across the polyhedrons? How do you determine the angle measurement? What evidence do you have for this pattern?* When teams are finished, have the move on to the "THINK ABOUT IT" at the end of Part B.

   - Joseph Plateau (1801-1883), a Belgian physicist, is famous for his "soapy" studies of shapes. He noticed, like you, that some simple patterns seemed to describe the geometry of how soap film surfaces come together in bounded shapes like the Zometool models. Look back at your observations in PART A and B. Summarize the patterns you found!
     - *Three surfaces of a soap film meet at 120 degree angles along an edge, or line.*
     - *Four lines, formed by the intersection of the three surfaces, meet at a vertex; the angle between the adjacent lines is approximately 109.5 degrees or the tetrahedral angle.*

   - Why didn't the soap film "cover" the faces of the triangular prism, the tetrahedron, and the cube?
     - *By merging toward the center, the soap film minimized its total surface area, and thus, the energy of the system.*

3. PART C: In this final portion, students have the opportunity to "free-play" with the Zometool models as they continue to explore patterns in soap films and bubbles. Several ideas of mini-investigations are provided in Part C. However, you may want to encourage students to come up with their own ideas for exploration as well. Following this investigation time, encourage students to share their patterns, the evidence for their patterns, and other interesting ideas or emerging questions. Add to the think board!



   Notes:
   - When you double-dip a polyhedron model in soap solution, you can often create an interior bubble. Alternatively, you can dip and wet a straw in soap solution and then insert the straw into the middle of any frame. Gently blow and you will create an interior bubble. So, it IS possible to create a cubic bubble – that is bounded in a Zometool model of a cube!

4. Investigation III: Examples of Informal Checks for Understanding

   - CLASS CHECK-IN: Return to the Think Board started in Investigation I. With sticky notes, prompt teams to add 3-4 pieces of new evidence, ideas, and/or questions. *What have we learned so far? What questions have emerged?*

   - INDIVIDUAL CHECK-IN (See Resource D):
     - Describe any connections you can make between the patterns of beehive honeycombs and the patterns of soap bubbles and films. *(Soap films surfaces and honeycomb cell walls meet at 120 degree angles)*
     - In the cube, we saw a rectangular soap film face in the center of the frame. Why didn't all of the soap film faces meet in the center, like the tetrahedron and the triangular prism? *(Only three soap films meet along an edge or line. In a cube, there are more than three films that would meet at the center.)*

- How was your thinking in this investigation similar to Belgium physicist Joseph Plateau and his discovery of soap film patterns in bounded shapes? *(e.g., finding patterns to discern and understand differing structures around us)* *This is a great strength to pull out and highlight in students!
- What was most surprising to you in this investigation?

Reflect and Extend: Nature's Patterns

ESSENTIAL QUESTION: *How do soap bubbles interact? How does nature choose its shapes?*

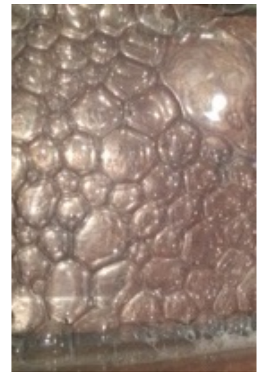
OVERVIEW: In this investigation, we will return to thinking about linkages between the patterns of honeycomb cells and the patterns of soap bubbles we began with in the Introductory Reason & Engage Prompts. Specifically, students will explore (a) what happens when soap bubbles meet, (b) connections between the structures of honeycombs and bubbles, and (c) reasons why nature often prefers hexagonal arrays.

TEACHER PREPARATION:

Suggested Instructional Arrangement: Teams of 2-4	**Zometool Kit (per team)** 13-SB **Materials:** Student Investigation Guide: Resource E One Small Tray per Team of Students Soap Solution (fill the trays ½ full with soap solution) Straws Graph Paper (one piece per student) Protractors

1. Distribute Resource E: Nature's Patterns, and refer students to PART A. Provide a brief overview of PART A. Walk around, listen, and observe. Key probing questions to ask teams are included in each of the steps below:

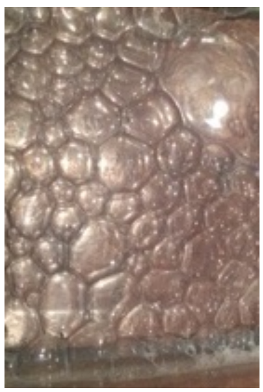
   #1: TEAM INVESTIGATE AND THINK: Prompt students to gently take turns gently blowing two bubbles that meet. These bubbles can be the same or differing sizes. Have them draw their observations in the three trial boxes. *What happens when two bubbles of equal size interact (they share a wall)? What happens if bubbles of different sides interact (the smaller bubble merges into the larger bubble)? What patterns are you finding?* Students are asked to think about why this happening as well *(soap bubbles share walls to minimize their total surface area – they are very economical).*

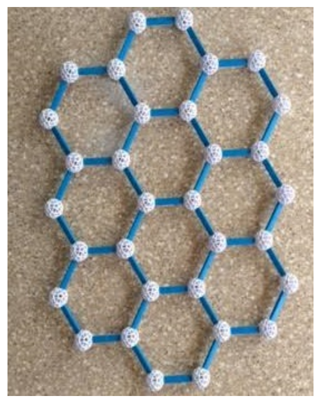


#2: TEAM INVESTIGATE AND THINK: Next, students should take turns gently blowing a cluster of bubbles. Have them draw their observations in the box provided and look for patterns in the bubble cluster arrangement. *Look carefully at the angles between the bubble walls. Do you recognize this angle (120 degree angles)? Can you measure this angle? What shapes are you seeing in the bubble cluster arrangement (bubbles are constantly changing shape to minimize their total surface area, but from a 2D perspective, students should start to notice a hexagonal arrangement)?* As teams finish, prompt them to move to question #3 individually or in teams.

#3: In this question, students are asked to think about the connections between the patterns of honeycombs and the patterns of soap bubble clusters *(share a common geometric pattern in nature: a network of 120 degree angles that create a hexagonal arrangement with shared walls and no gaps)*

*It might be helpful to have a Zometool model of honeycomb here!

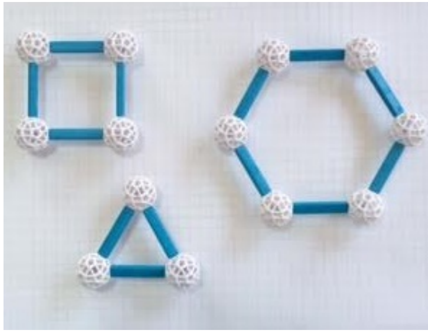


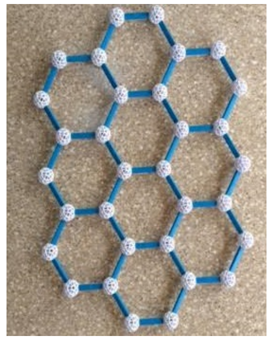


2. PART B: In this mini-investigation, students explore reasons for the hexagonal array in nature. Point out to students that there are only three regular polygons that can tile a flat surface with no gaps or overlaps: the triangle, the square, and the hexagon. *So, why do bees use the hexagon as their design choice?* Let's build some honeycomb cells and find out! Be sure that each team has 13 short blue struts. Tell students that each strut represents honeycomb wall material (that is very expensive for the bees to make).

   TEAM BUILD: Each team should build a Zometool model of a regular triangle, square, and hexagon. *What are the interior angles of each?* Show students the Zometool model of the honeycomb so they clearly make the connection between hexagons, hexagonal arrays, and the 120 degree angle.

   INDIVIDUAL & TEAM THINK: Provide every student a piece of graph paper. Prompt teams to work on showing why bees choose to use hexagonal cells, over regular triangles and squares. *If teams are struggling, challenge them to think about the area inside the polygon (count the squares that represent living and storage space) vs. the perimeter (number of struts or the amount of honeycomb cell wall material needed to enclose this space). They will find that the hexagon maximizes the area enclosed in the honeycomb (i.e., space for storage) while minimizing the perimeter (i.e., least amount f wax needed to construct the honeycomb cell walls.





When teams are finished, have each individual student develop an explanation for why bees choose to use hexagonal cells – in their own words and with their own representations.

3. (Optional): HEXAGONS IN NATURE. Have students choose a structure of interest with a hexagonal array to explore (e.g., the scutes of a turtle carapace, the facets of a fly's eye, mud cracks, a snowflake). *Why does ___________________ have a hexagonal array in its design? How is this similar and/or different than the soap bubbles and honeycomb?*

4. PART C: Checks for Understanding

   - CLASS CHECK-IN: Return one more time to the Think Board started in Investigation I. With sticky notes, prompt teams to add 3-4 pieces of new evidence, ideas, and/or questions. *What did we learn in this investigation? How did our ideas and thinking change over time? Why did they change? What else could we explore?*

   - INDIVIDUAL CHECK-IN (Resource E):
     - Why do you think bubble clusters also form hexagonal arrangements? S*oap bubbles change shape, merge, and share walls also to maximize their space filling capacity while using the "least amount of material". The 120 degree angle is nature's way of doing the most with the least!*

     - How does nature choose its shapes? How is the ability to find and "see" patterns helpful in understanding the structures around you? *These questions offer students a chance to summarize their ideas, ways of thinking, and discoveries!*

Name(s)____

PART I: Defining a Pattern

A pattern is...	A pattern is not..
PATTERN	
This is an example of a pattern.	This is not an example of a pattern.

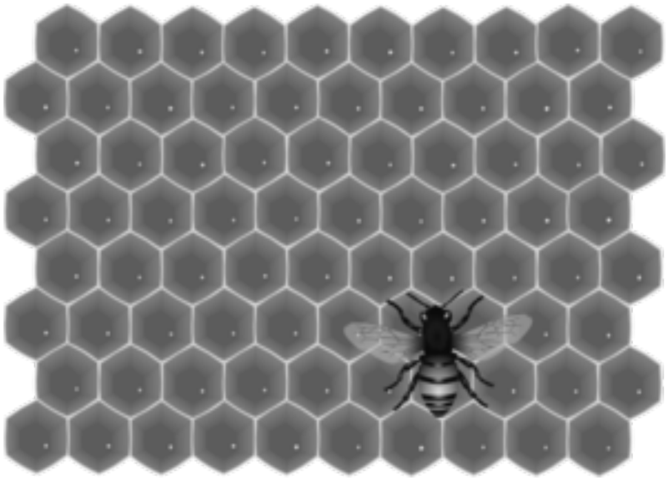
A pattern is:

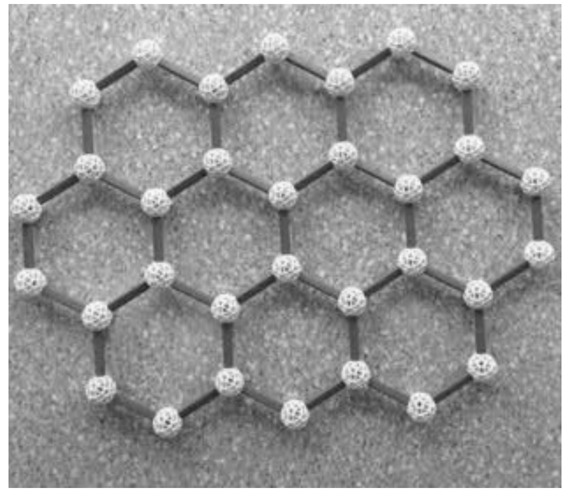
____

____

____.

PART II: Seeing Patterns in Nature's Structures: Beehive Honeycomb





DID YOU KNOW? The walls of a honeycomb cell are made of beeswax. Making beeswax is very expensive process for the bees and requires a tremendous amount of energy. Thus, bees are careful to not waste this resource when creating cells that will store honey, nectar, and pollen.

1. What patterns do you see in the design of a beehive honeycomb?

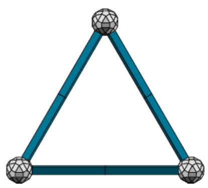
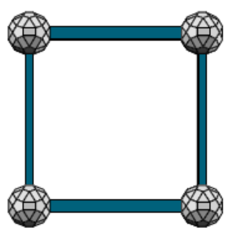
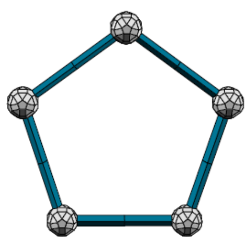
2. What do you think might be the reason(s) for this honeycomb design? Why not make the walls circular, or even triangular?

Name(s)____

Exploring the Patterns of Soap Film & Bubble Structures

Part I:

<u>PREDICT</u>: For each "polygon wand", draw a prediction of what you think the bubble will look like.

Polygon Wand		Predicted Bubble Shape	Actual Bubble Shape
TRIANGLE			
SQUARE			
PENTAGON			

<u>OBSERVE</u>: Dip the polygon gently into and out of the soap solution. Try to blow at least 2-3 bubbles and observe. *What patterns can you find?* Draw the resulting bubble shape!

<u>EXPLANATION</u>: Explain *what* happened and *why* you think it happened.

EXPLANATION CHECKER: Did I…

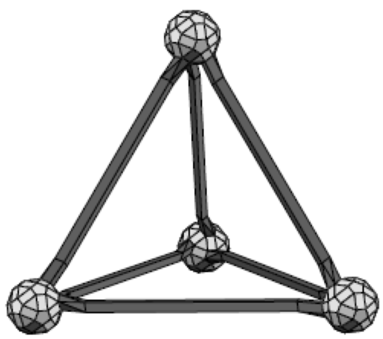
- ☐ Summarize the pattern(s) I found?
- ☐ Explain how I found this pattern?
- ☐ Show this pattern in at least two ways (written, diagram, picture, etc.)
- ☐ Explain WHY this happened?

Name(s)____

Exploring Soap Film Patterns in 3D Shapes

PART A

PREDICT: Imagine that you dip this tetrahedron in a soap solution. What will the soap film look like? Draw your prediction!



TETRAHEDRON: 6-MG

I think this because:

My Observations:

How many soap film faces come together to make a line or edge?

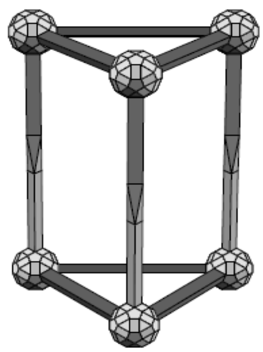
How many lines come together at a vertex?

What is the measure of the angles where you see the films join?

*Using a white Zometool node as a "soap film vertex" and the yellow struts to represent the lines that come together at this vertex, build a model that represents this junction. *Look carefully at the angles here.*

EXPLANATION – Why do you think that this happened?

PART B: Dip the following polyhedrons in the soap solution. Continue to look for patterns in these soap structures!



TRIANGULAR PRISM
6-MB & 3-LY

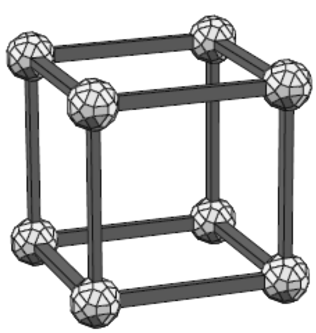
My Observations:

How many soap film faces come together to make a line or edge?

What is the measure of the angles where you see the films join?

How many lines come together at a vertex?

What is the measurement of the angles between the lines coming into the vertex?



CUBE: 12-MB

My Observations:

How many soap film faces come together to make a line or edge?

What is the measure of the angles where you see the films join?

How many lines come together at a vertex?

What is the measurement of the angles between the lines coming into the vertex?

THINK ABOUT IT:

1. Joseph Plateau (1801-1883), a Belgian physicist, is famous for his "soapy" studies of shapes. He noticed, like you, that some simple patterns seemed to describe the geometry of how soap film surfaces come together in bounded shapes like the Zometool models. Look back at your observations in PART A and B. Summarize the patterns that you found!

2. Why do you think that the soap films did not "cover" the faces of the triangular prism, the tetrahedron, and the cube?

PART C: With the Zometool frames, try some of these mini-investigations. Describe the patterns you find!

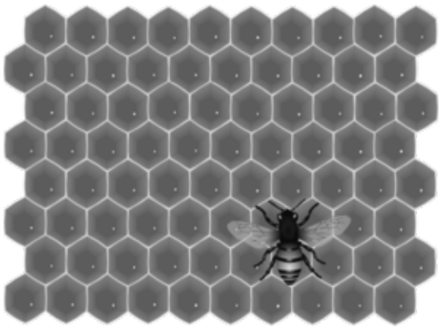
- Double-dip the polyhedrons and try to blow or catch a bubble inside! What do you notice about the shape of the bubble?

- Touch the soap films with a wet (with soap solution) and a dry finger? What happens?

- What happens if you start to poke out the different soap film faces?

Name____

1. Describe any connections you can make between the patterns of beehive honeycombs and the patterns of soap bubbles and films.



2. In the cube, we saw a rectangular soap film face in the center of the frame. Why didn't all of the soap film faces meet in the center, like the tetrahedron and the triangular prism?

3. How was your thinking in this investigation similar to Belgium physicist Joseph Plateau and his discovery of soap film patterns in bounded shapes?

4. What was most surprising to you in this investigation?

Name(s)____

Exploring Nature's Patterns

PART A: *What happens when two or more bubbles meet/interact?*

1. Using the soap solution and straws provided, blow two bubbles that touch. The bubbles can be different sizes. Draw your observations below. Then, try this two more times!

TRIAL 1	TRIAL 2	TRIAL 3

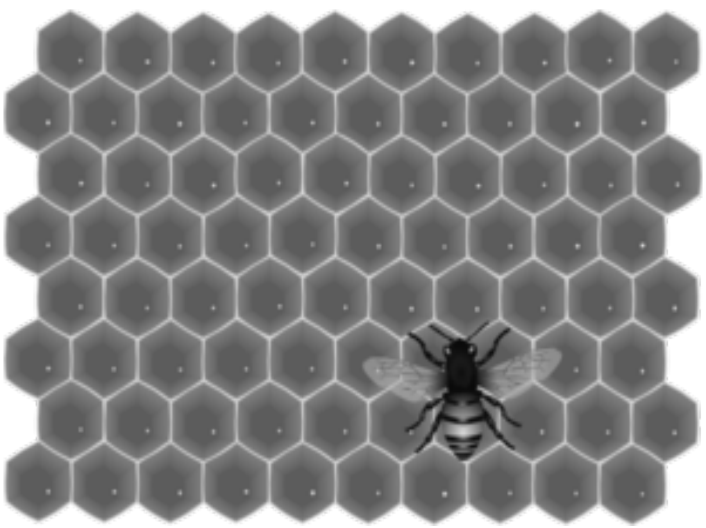
What happens when two bubbles meet or interact? What patterns do you see?

Why do you think that this happens?

2. Now, gently blow lots of joined bubbles, or a bubble cluster. Draw your observations below.

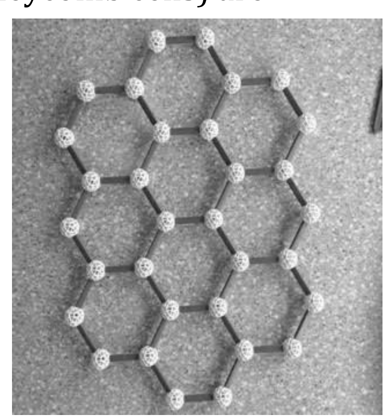
Describe any patterns that you see in how the bubbles arrange themselves. (Hint: Look carefully at angles between the bubble walls – use your protractor!).

3. What connections can you make between the patterns of honeycombs and the patterns of soap bubble clusters?



PART B: *Why does nature often prefer hexagonal arrays?*

LET'S INVESTIGATE: Remember that the walls (or the perimeter of the honeycomb cells) are very expensive for bees to make. So, w*hy do you think that the bees make their cells using hexagons instead of equilateral triangles or squares?* After all, these last two shapes also "tile" or cover a flat surface with no gaps or overlaps.



a) Build a Zometool model of a regular triangle, square, and hexagon using only the short blue struts. Imagine that these struts represent honeycomb "wall material."

b) Using the graph paper provided, try to show why the bees have chosen to use hexagonal cells, instead of triangular or square cells.

EXPLANATION (Individual): *Why do the bees make their cells using hexagons? Be sure to share your thinking, reasoning, and diagrams.*

PART C:

4. Why do you think bubble clusters also form a hexagonal array?

5. For each of the two prompts below, summarize 2-3 big or "walk-away" ideas that you have learned from exploring the geometry of soap films and bubbles, and connections to the beehive honeycomb!

HOW DOES NATURE CHOOSE ITS SHAPES?

(Why are free-floating bubbles spherical? Why do soap bubbles and honeycombs share walls, and join at 120 degree networks of angles? What do these all have in common?)

HOW IS THE ABILITY TO FIND AND "SEE" PATTERNS HELPFUL IN UNDERSTANDING THE STRUCTURES ALL AROUND YOU?